



Note: Not more than Three Questions to be Attempted

Q.1: A particle is moving along a straight line with the acceleration

$$a = (9\sqrt{t} - 16t + 1), \text{ m/s}^2$$

where  $t$  is in seconds. Determine the velocity and the position of the particle as a function of time. When  $t = 0, v = 0$  and  $s = 20 \text{ m}$ .

Solution:

$$\begin{aligned} - a &= \frac{dv}{dt} \Rightarrow adt = dv \\ \text{or } \int_0^v dv &= \int_0^t adt \Rightarrow \int_0^v dv = \int_0^t (9\sqrt{t} - 16t + 1) dt \\ \therefore v &= 6t^{\frac{3}{2}} - 8t^2 + t \quad \underline{\text{Ans. 1}} \end{aligned}$$

$$\begin{aligned} - v &= \frac{ds}{dt} \Rightarrow vdt = ds \\ \int_{20}^s ds &= \int_0^t (18t^{\frac{3}{2}} - 8t^2 + t) dt \\ s - 20 &= \frac{12}{5}t^{\frac{5}{2}} - \frac{8}{3}t^3 + \frac{1}{2}t^2 \end{aligned}$$

$$\text{or } s = \frac{12}{5}t^{\frac{5}{2}} - \frac{8}{3}t^3 + \frac{1}{2}t^2 + 20 \quad \underline{\text{Ans. 2}}$$

**Q.2:** The system shown in the figure is released from rest with cable taut. For the friction coefficients  $\mu_s = 0.25$  (static) and  $\mu_k = 0.20$  (kinetic), calculate the acceleration of each body and the tension in the cable. Neglect the small mass and friction of the pulleys. (Note: Assume  $g = 9.81 \text{ m/s}^2$ ).

**Solution:**

- check Direction of Motion

$$2T_s = 196.2 \text{ N} \Rightarrow T_s = 98.1 \text{ N}$$

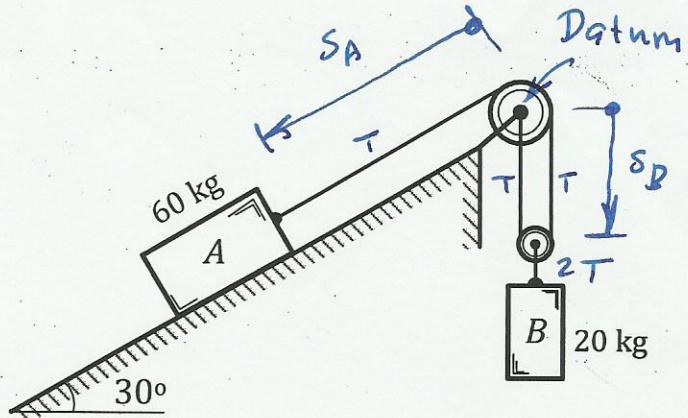
$$294.3 - 98.1 = 192.36 \text{ N} \leftarrow$$

$$192.36 \text{ N} > 0.25 \times 509.7 \text{ N}$$

$$> 101.9 \text{ N (Dynamic)}$$

and

$$192.36 \text{ N} > 0.2 \times 509.7 \text{ N (Motion Downward)}$$



- Equations of Motion

From F.B.D. ① :

$$\sum F_x = m \cdot a_A$$

$$294.3 - 101.9 - T = 60 a_A \quad \text{---} \textcircled{1}$$

From F.B.D. ② :

$$\sum F_y = m \cdot a_B$$

$$196.2 - 2T = 20 a_B \quad \text{---} \textcircled{2}$$

- Kinematics :

$$SA + 2SB = l, \text{ and}$$

$$a_A + 2a_B = 0 \Rightarrow a_B = -\frac{1}{2}a_A$$

$$\text{or } a_A = -2a_B \text{ (Sub. into } \textcircled{1} \text{ or } \textcircled{2})$$

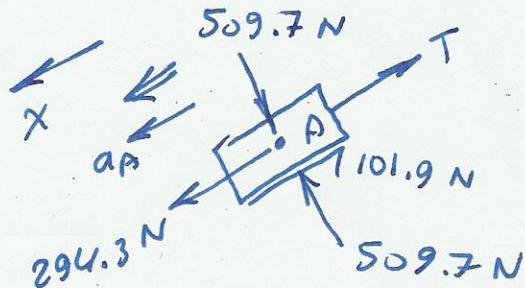
Solve the two equations, yields :

$$a_A = 1.45 \text{ m/s}^2 \leftarrow \underline{\text{Ans.}}$$

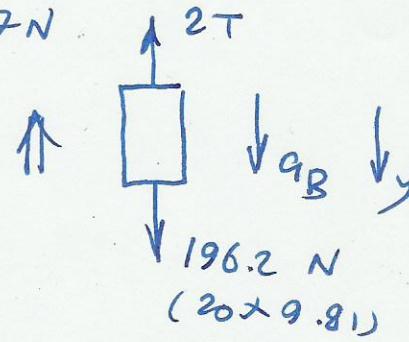
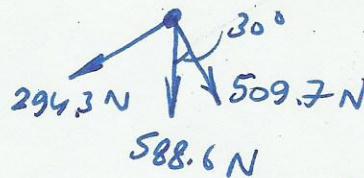
$$a_B = -0.725 \text{ m/s}^2 = 0.725 \uparrow \text{ m/s}^2 \underline{\text{Ans.}}$$

and

$$T = 105.4 \text{ N} \quad \underline{\text{Ans.}}$$



F.B.D. ①



F.B.D. ②

**Q.3:** A projectile shown in the figure is fired from the edge of a 150 m cliff with an initial velocity of 180 m/s, at an angle of  $30^\circ$  with the horizontal. Neglecting air resistance, find the horizontal distance from the gun to the point where projectile strikes the ground. (Note: Assume  $g = 9.81 \text{ m/s}^2$ ).

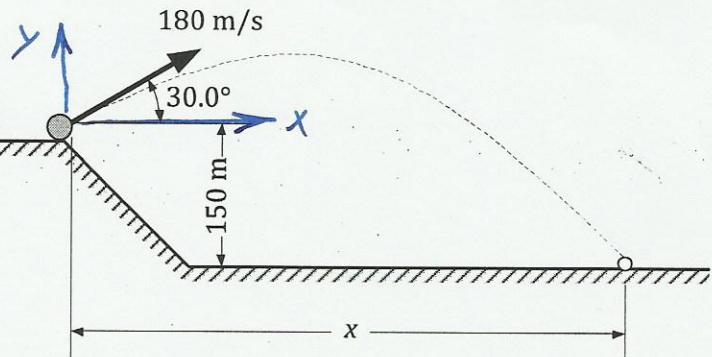
**Solution :**

$$v_{x_0} = 180 \cos 30^\circ = 155.9 \text{ m/s}$$

$$v_{y_0} = 180 \sin 30^\circ = 90 \text{ m/s}$$

$$y = v_{y_0} t + \frac{1}{2} a t^2$$

$$(a = -g = -9.81 \text{ m/s}^2)$$



$$y = -150 \text{ m}$$

$$\therefore -150 = 90t - \frac{1}{2}(9.81)t^2$$

or

$$4.905t^2 - 90t - 150 = 0$$

$$t = 19.87 \text{ s} \quad \text{and} \quad t = -1.54 \text{ s} \quad (\text{neglected})$$

$$x = v_{x_0} t \Rightarrow x = 155.9(19.87) = 3098 \text{ m}$$

$$\text{say } x = 3100 \text{ m} \quad \underline{\text{Ans.}}$$

Q.4: If the coefficient of kinetic friction between the 100 kg crate and the plane, shown in the figure, is  $\mu_k = 0.30$ . Use the principle of work and energy to determine the speed of the crate at the instant the compression of the spring is  $x = 1.25 \text{ m}$ . Initially the spring is unstretched and the crate is at rest. (Assume  $g = 9.81 \text{ m/s}^2$ ).

Solutions :

$$N = 100 \times 9.81 \cos 45^\circ \\ = 693.7 \text{ N}$$

$$F_f = 0.3(693.7) \\ = 208.1 \text{ N}$$

$$F_s = kx = 2000x$$

$$h = (12 + 1.25) \sin 45^\circ \\ = 8.57 \text{ m}$$

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 981 \times 8.57 - 208.1(12 + 1.25) \\ - \frac{1}{2}(2000)(1.25)^2 = \frac{1}{2}(100)v^2$$

$$\therefore v = 9.0 \text{ m/s} \quad \underline{\text{Ans.}}$$

